INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2022-23
Statistics - III, Mid-semesteral Examination, September 22, 2022
Marks are shown in square brackets. **Total Marks: 50**
Time: $2\frac{1}{2}$ Hours

You may use any of the results stated and discussed in the lecture notes, by stating them explicitly. Results from the assignments may not be used without establishing them.

1. Suppose (X, Y) is bivariate normal with $E(X) = 0 = E(Y)$, $Var(X) = \sigma^2 = Var(Y)$ and a correlation coefficient of $\rho = 0.4$ between X and Y.
(a) Find the distribution of

$$Q = X^2 + \frac{(Y - \rho X)^2}{1 - \rho^2}.$$

(b) Find $E(Q)$. [10+3]

2. Suppose $\mathbf{Y} \sim N_p(\mathbf{0}, \mathbf{\Sigma})$.
(a) Let $\Sigma = \sigma^2 I_p$, and consider two symmetric idempotent $p \times p$ matrices P_1 and P_2 such that $P_1 P_2 = P_2 = P_2 P_1$. If $\mathbf{U} = P_2 \mathbf{Y}$ and $\mathbf{V} = (P_1 - P_2)\mathbf{Y}$, find the conditional distribution of $\mathbf{U}$ given $\mathbf{V} = \mathbf{v}$.
(b) Let Σ be p.s.d. and $B_{p\times p}$ be symmetric. Show that $\mathbf{Y}'B\mathbf{Y}$ has the χ^2 distribution if and only if $\Sigma B \Sigma B \Sigma = \Sigma B \Sigma$. [10+11]

3. Consider the following model:
$y_1 = \theta + 2\phi + \epsilon_1$
$y_2 = \theta + \phi + \gamma + \epsilon_2$
$y_3 = \phi - \gamma + \epsilon_3$
$y_4 = 2\phi - 2\gamma + \epsilon_4$,
where θ, ϕ, γ are unknown constants and ϵ_i are uncorrelated random variables having mean 0 and variance σ^2.
(a) Show that $\gamma - \phi$ is estimable. What is its BLUE?
(b) What is the degrees of freedom for the residual sum of squares?
(c) Provide an expression for an unbiased estimate of σ^2. (Computations are not required.) [12+2+2]